Joint Routing and Charging Problem of Electric Vehicles

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Outline

1. Research Background
2. Electric Vehicles Routing Problem
3. Electric Vehicles Routing Problem with Time Flexibility
4. Conclusion

Outline

1. **Research Background**
2. Electric Vehicles Routing Problem
3. Electric Vehicles Routing Problem with Time Flexibility
4. Conclusion

Research background

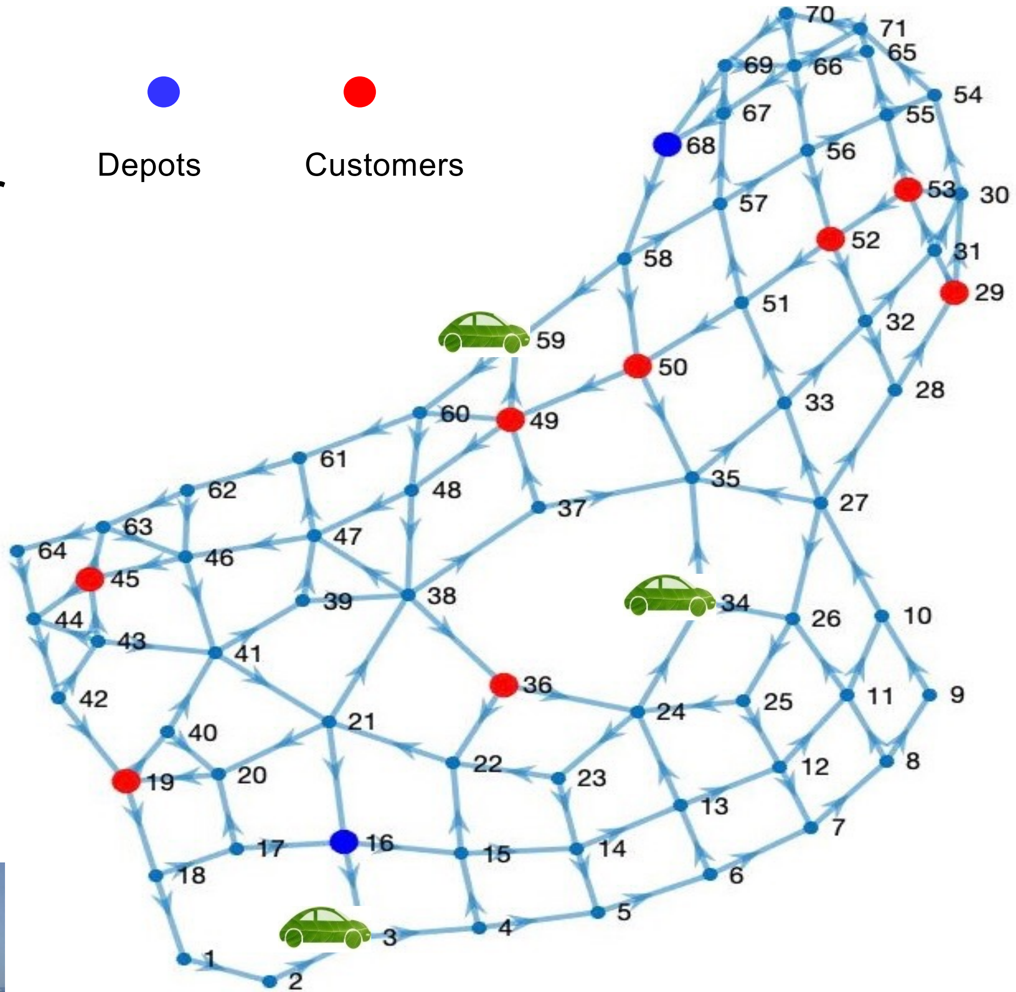
❑Transportation Problem

➢Definition

- Design the optimal routes for a fleet of vehicles, to serve the customers.

➢Applications

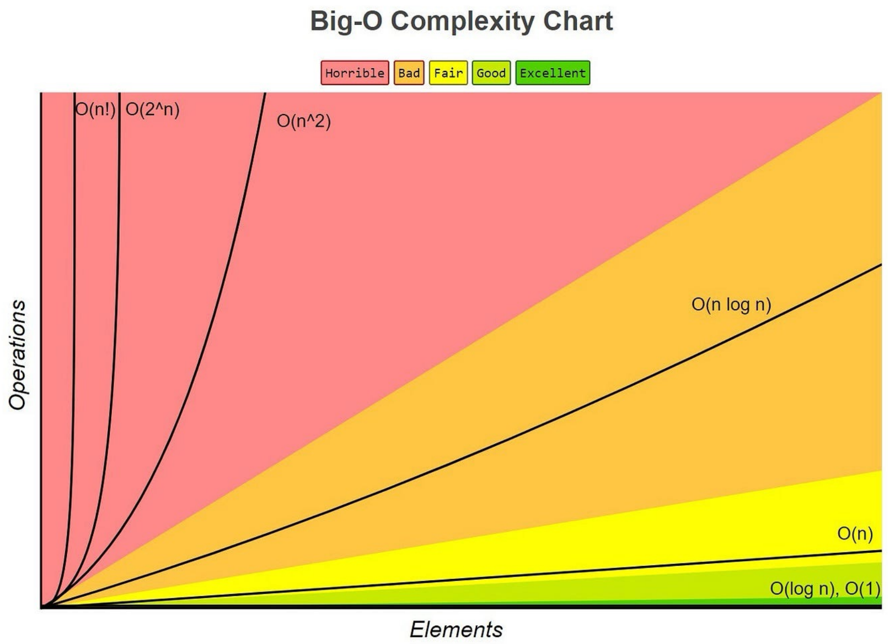
- Package delivery, and taxi operation

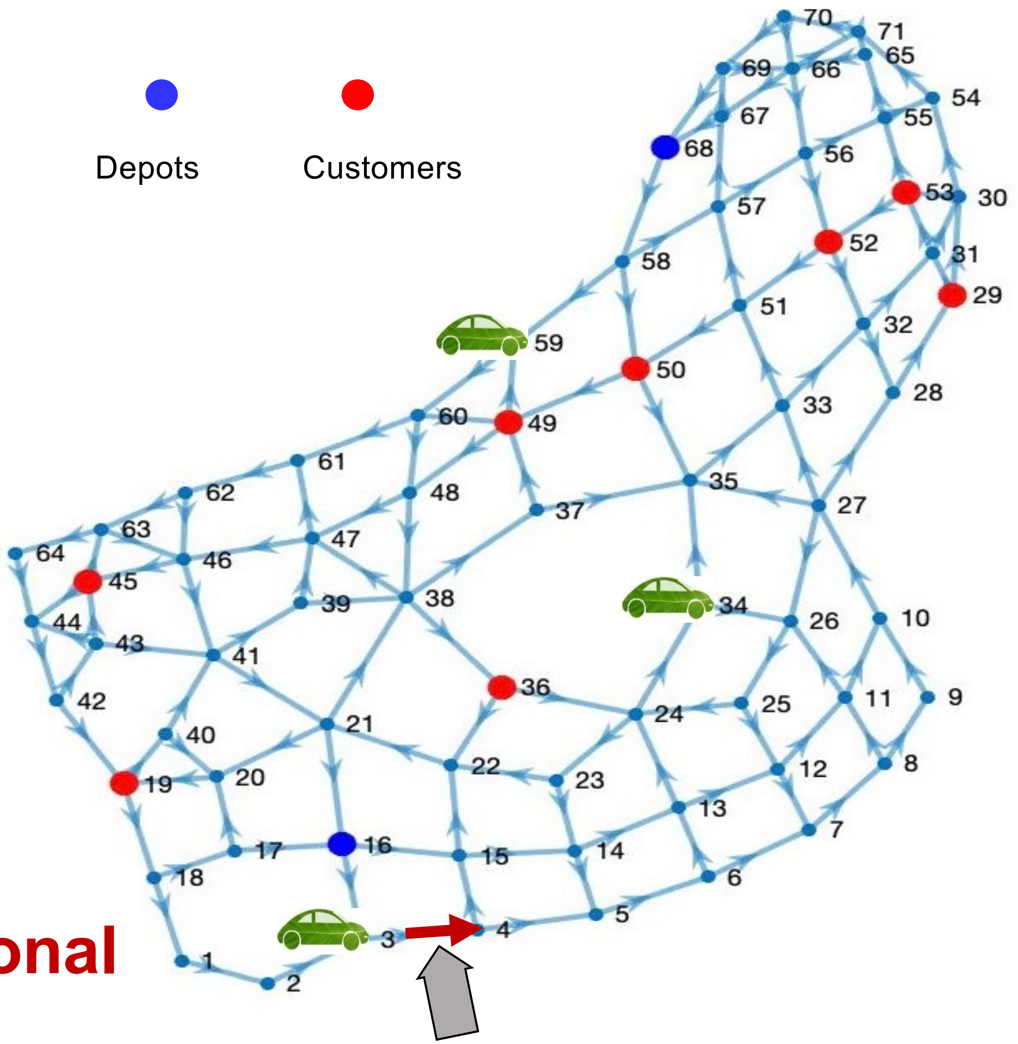


Vehicle routing problem

Research background

❑Vehicle routing problem





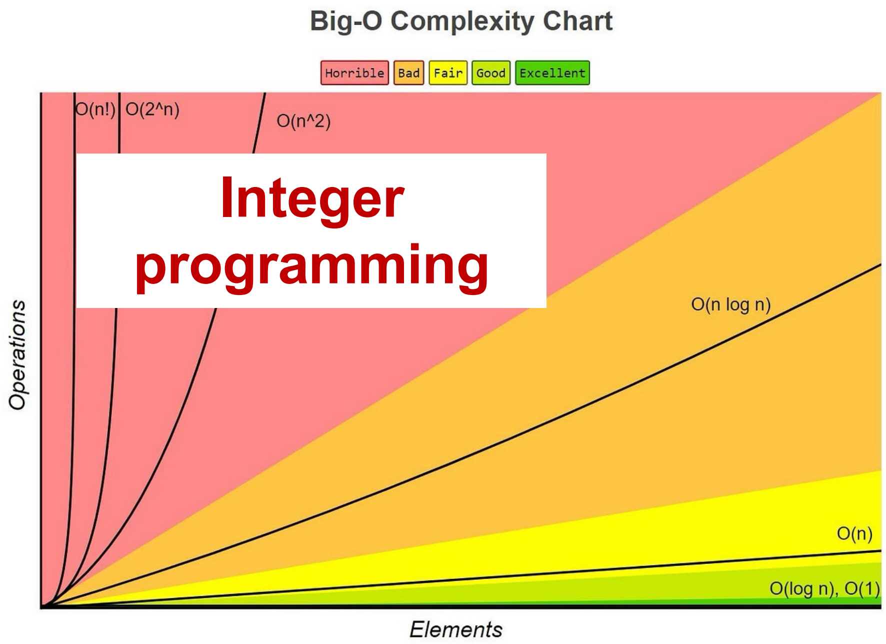
High computational complexity!

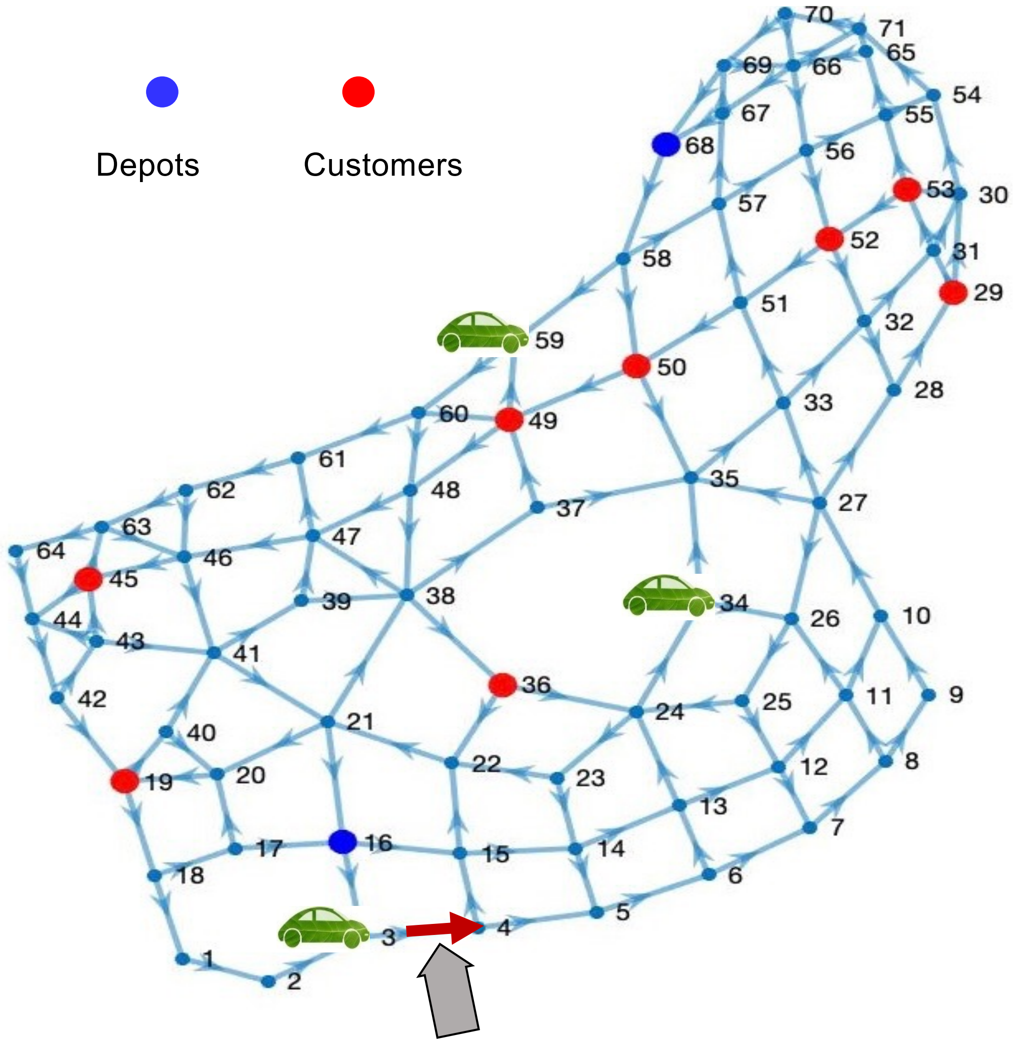
$x_{ij}^{k} \in \{0,1\}$

[1] Toth, Paolo, and Daniele Vigo. Vehicle routing: problems, methods, and applications. SIAM, 2014.

Research background

❑Vehicle routing problem





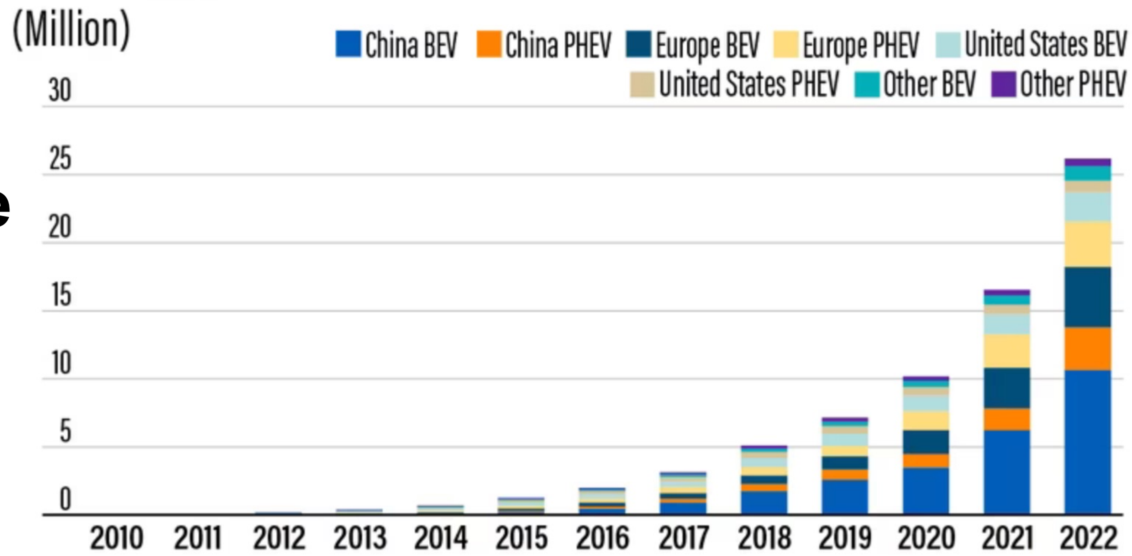
$$x_{ij}^{k} \in \{0,1\}$$

[1] Toth, Paolo, and Daniele Vigo. Vehicle routing: problems, methods, and applications. SIAM, 2014.

Research background

❑Electric vehicle

- ➢Lower emissions
- ➢Reduced dependence on fossil resources



❑Limitations

- ➢Range anxiety
- ➢Long charging time

Vehicle routing problem ➡ **Electric vehicle routing problem**

Research background

❑Electric vehicle routing problem (EVRP)

➢**VRP**

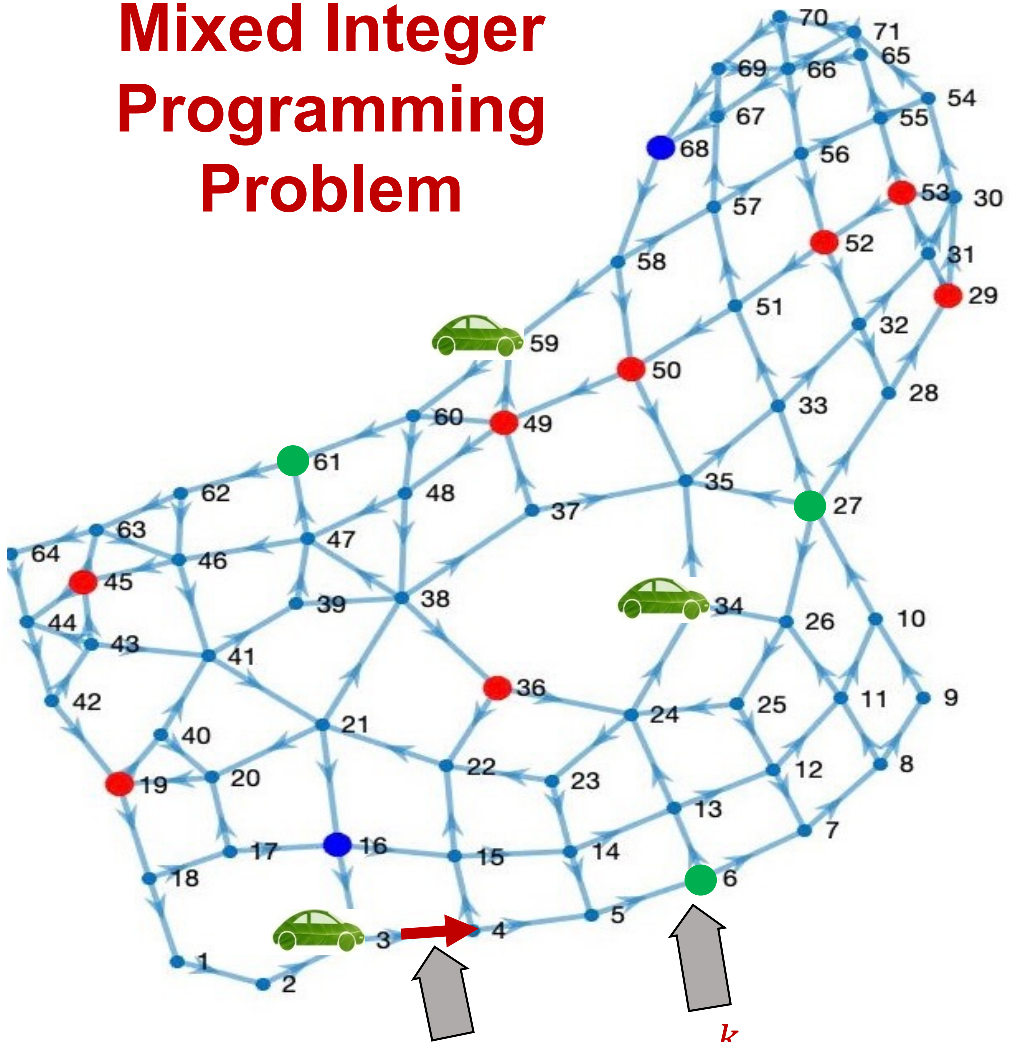
➢**Objective**

- Usage cost, travel time

➢**Constraint**

- Vehicle flow
- Time window

Mixed Integer Programming Problem



$x_{ij}^{k} \in \{0,1\}$ r_i^k

Research background

❑Literature review

➢Exact algorithms

- Branch and bound method (TITS'18)
- Branch, cut and price method (IJPR'22, TS22)
- Solver-based method (TSG'18, TVT'20)

High computation complexity

➢Approximate approaches

- Dual decomposition (TSG'18, TVT'20)
- ADMM (TSG'18, TVT'20)

Suboptimal, high computation complexity

➢Heuristics methods

- Variable neighborhood search (TSG'18, TVT'20)
- Genetic algorithm (TSG'18, TVT'20)
- Deep reinforcement learning method (TSG'18, TVT'20)

Subotimal

Research background

❑Motivation

➢Large-sized electric vehicle routing problem

- The compuataion time of large-sized EVRP, is quite long!
- The online EVRP requires a computation-efficient algorithm!

➢Collaborative scheduling of electric vehicle fleet and customers

- Fixed pickup time of customers reduces the efficiency of electric transportation system.
- To characterize the collaborative scheduling problem, how to formulate a suitable mathematical model?

Outline

1. Research Background

2. **Electric Vehicles Routing Problem**

3. Electric Vehicles Routing Problem with time flexibility

4. Conclusion

EV routing problem

❑Mathematical model (MIP)

➢Objective function

$$\min_{x_{ij}^k, r_i^k} \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} \begin{pmatrix} c_i + \omega_T T_{ij} \\ + r_i^k p_i + \omega_T r_i^k g_i \end{pmatrix} x_{ij}^k$$

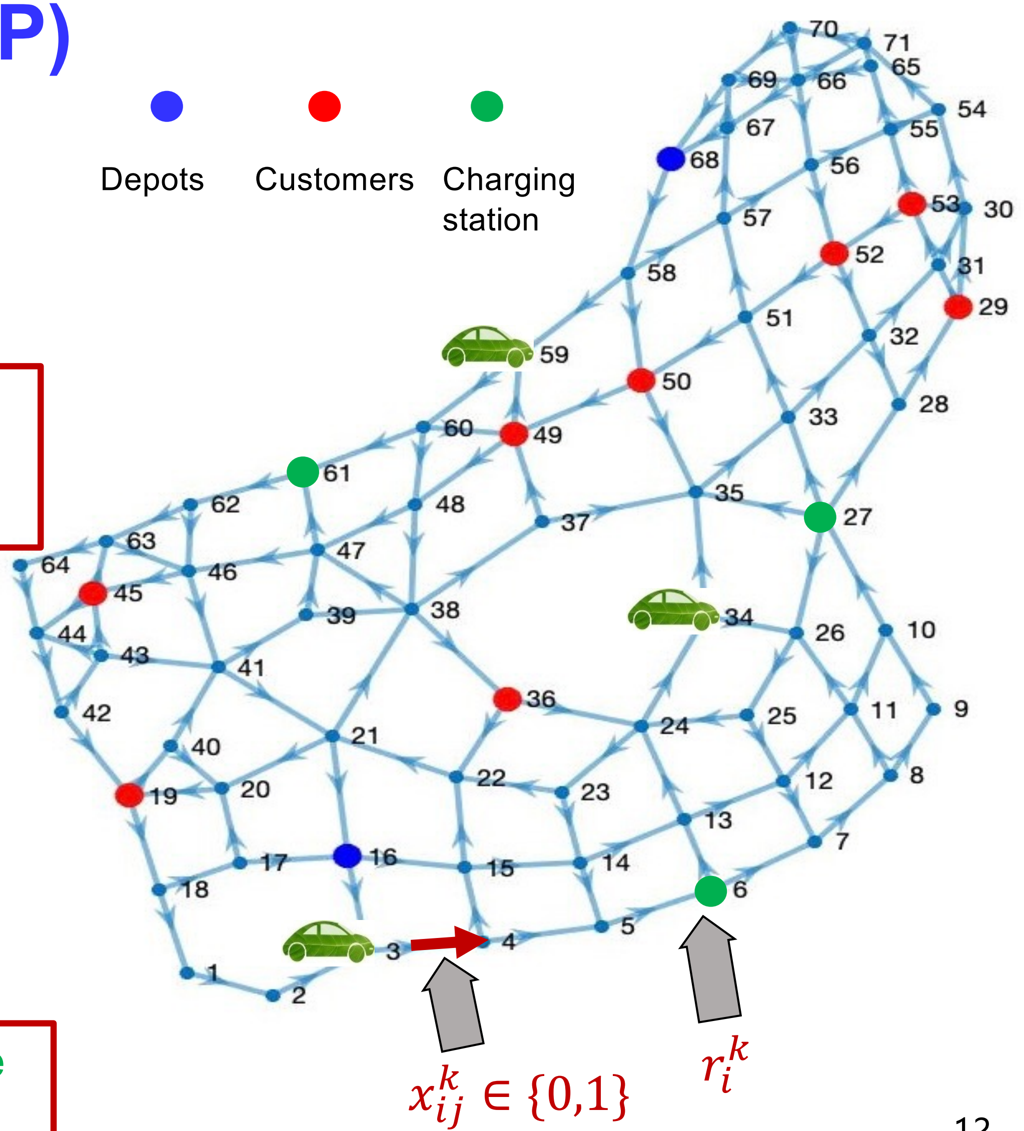
Operation cost = usage cost +travel time
+ charging cost
+charging time

➢Constraints

- Vehicle flow

$$\sum_{j\in\mathcal{V}} x_{ij}^k - \sum_{j\in\mathcal{V}} x_{ji}^k = b_i, b_{v_1} = 1, b_{v_n} = -1, b_i = 0,$$

Starting from the start depot, after visiting the customers, return to the end depot



EV routing problem

❑Mathematical model (MIP)

➢Constraints

- Visiting constriant

$$\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{V}} x_{ij}^{k} \leq 1,$$

Customer cannot be visited by more than one EV

- Visiting time

$$t_j \geq \left(T_{ij} + g_i r_i^k + t_i\right) x_{ij}^k,$$

The visiting time of customers

- SoC of EV

$$E_j^k = \sum_{i\in\mathcal{V}} \left(E_i^k + r_i^k - e_{ij}\right) x_{ij}^k,$$

$$0 \leq E_j^k \leq \bar{E}.$$

The state of charge of EV

EV routing problem

❑Mathematical model (MIP)

$$\min_{x_{ij}^k \in \mathbb{B}, r_i^k \in \mathbb{R}} \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} \left(c_i + \omega_T T_{ij} + r_i^k p_i + \omega_T r_i^k g_i\right) x_{ij}^k$$

s.t.

$$\sum_{j\in\mathcal{V}} x_{ij}^k - \sum_{j\in\mathcal{V}} x_{ji}^k = b_i, \quad \forall i \in \mathcal{V}, k \in \mathcal{K}, b_{v_1} = 1, b_{v_n} = -1, b_i = 0$$

$$\sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{V}} x_{ij}^k \le 1, \quad \forall i \in \mathcal{R}$$

$$t_j \ge \left(T_{ij} + g_i r_i^k + t_i\right) x_{ij}^k, \forall i \in \mathcal{V} \setminus v_n, j \in \mathcal{V} \setminus v_1, k \in \mathcal{K}$$

$$E_j^k = \sum_{i\in\mathcal{V}} \left(E_i^k + r_i^k - e_{ij}\right) x_{ij}^k, \forall \mathrm{j} \in \mathcal{V} \setminus v_1, \mathrm{k} \in \mathcal{K},$$

$$0 \le E_j^k \le \bar{E}, \forall \mathrm{j} \in \mathcal{V} \setminus v_1, \mathrm{k} \in \mathcal{K}.$$

EV routing problem

❑Mathematical model (MIP)

$$\min_{x_{ij}^k \in \mathbb{B}, r_i^k \in \mathbb{R}} \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} (c_i + \omega_T T_{ij} \qquad) x_{ij}^k$$

s.t.

$$\sum_{j \in \mathcal{V}} x_{ij}^k - \sum_{j \in \mathcal{V}} x_{ji}^k = b_i, \quad \forall i \in \mathcal{V}, k \in \mathcal{K}, b_{v_1} = 1, b_{v_n} = -1, b_i = 0$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{V}} x_{ij}^k \le 1, \quad \forall i \in \mathcal{R}$$

$$t_j \ge \left(T_{ij} + g_i r_i^k + t_i\right) x_{ij}^k, \forall i \in \mathcal{V} \setminus v_n, j \in \mathcal{V} \setminus v_1, k \in \mathcal{K}$$

Theorem 1 (**Dimension reduction**)

Primal problem

Elimination of bilinear terms

Exact LP relaxation

Routing problem

Integer solution from the routing problem

Charging scheduling

EV routing problem

❑Two stage method (Routing problem)

IP → LP

- **Elimination of bilinear terms**
  - Eliminating bilinear term $r_i x_{ij}$ (r_i is replaced with e_{ij})
- **Exact LP relaxation**
  - Vehicle flow constraints. are totally unimodular, satisfying sufficient condition of exact LP relaxation
  - Time constraint does not break LP relaxation exactness

$$\min_{x_{ij}\in\{0,1\}} \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}}(c_i+\omega_T T_{ij})x_{ij}$$

$$\text{s.t.}\quad t_j \ge (T_{ij}+t_i+g_i r_i)x_{ij}, \forall i\in\mathcal{V}\setminus v_n,\ j\in\mathcal{V}\setminus v_1$$

$$\sum_{j\in\mathcal{V}} x_{ij} - \sum_{j\in\mathcal{V}} x_{ij} = b_i, \quad \forall i\in\mathcal{V},$$

$$b_{v_1}=1, b_{v_n}=-1, b_i=0$$

$$\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{V}} x_{ij} \le 1, \quad \forall i\in\mathcal{R}$$

Primal problem → (Elimination of bilinear terms / Exact LP relaxation) → **Routing problem** → (Integer solution from the routing problem) → Charging scheduling

EV routing problem

❑Two stage method (Charging scheduling)

- **Given x_{ij} , obtain the path P_k for each EV**

How to get a better solution?

$$\min_{r_i \in R, i \in P_k} \quad \omega_T \sum_{i \in P_k} r_i g_i + \sum_{i \in P_k} r_i p_i$$

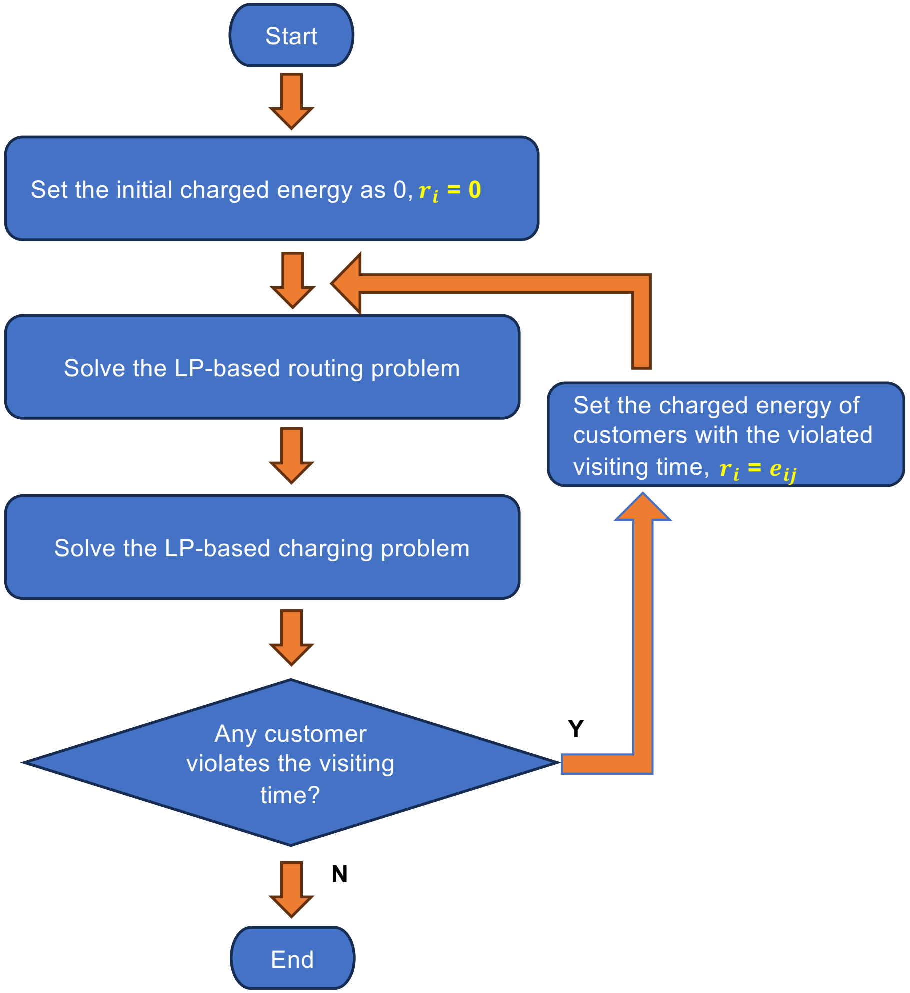
$$\text{s.t.} \quad E_j = E_i + r_i - e_{ij}, \forall j \in P_k \setminus v_1$$

$$0 \leq E_j^k \leq \bar{E}$$

The charging scheduling problem is a **LP** problem

EV routing problem

❑Iterative two stage method



❑Performance guarantee

- **Two stage method**
  - Both routing and charging problem are LP problems, which reduces the computation time
- **Iterative method**
  - Converge in finite iterations
- **Optimality gap**

$$C_{MIP} \leq C_{ILP} \leq C_{TLP}$$

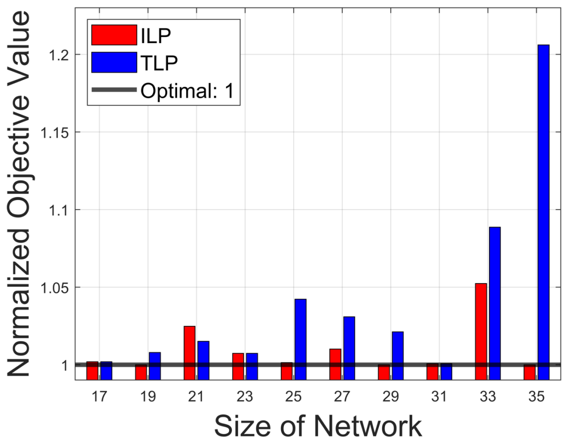
- **Computation time**

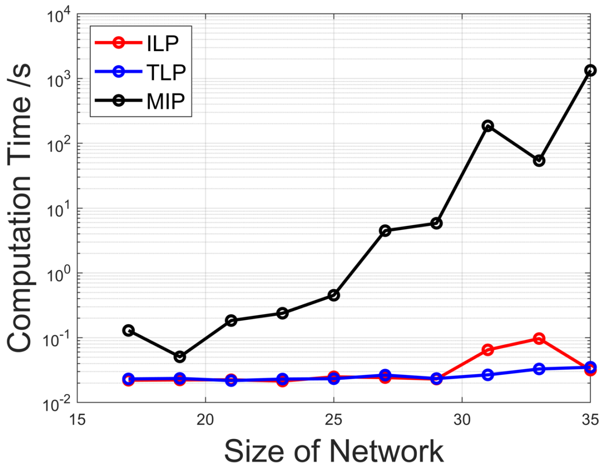
$$T_{MIP} \gg T_{ILP} \geq T_{TLP}$$

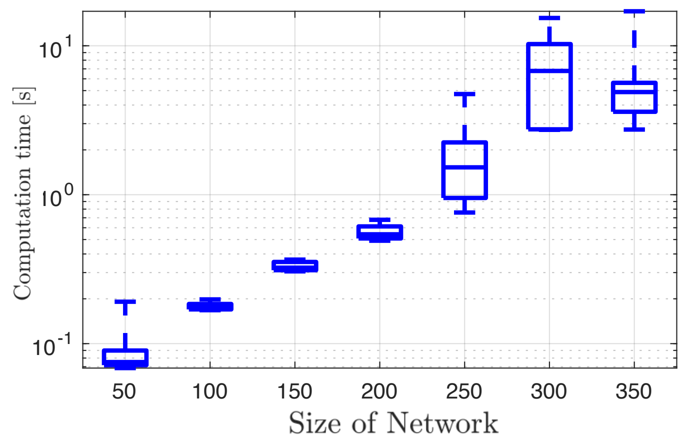
EV routing problem

❑Simulation results

- ➢**Comparison between TLP, and ILP**
- ➢**Large-sized problem**







The optimality gap of ILP lower than 10%
The optimality gap of TLP lower than 20%

The computation time of ILP 0.1 s
The computation time of TLP 0.05 s

350 nodes and
35 EVs solved
in 10 seconds

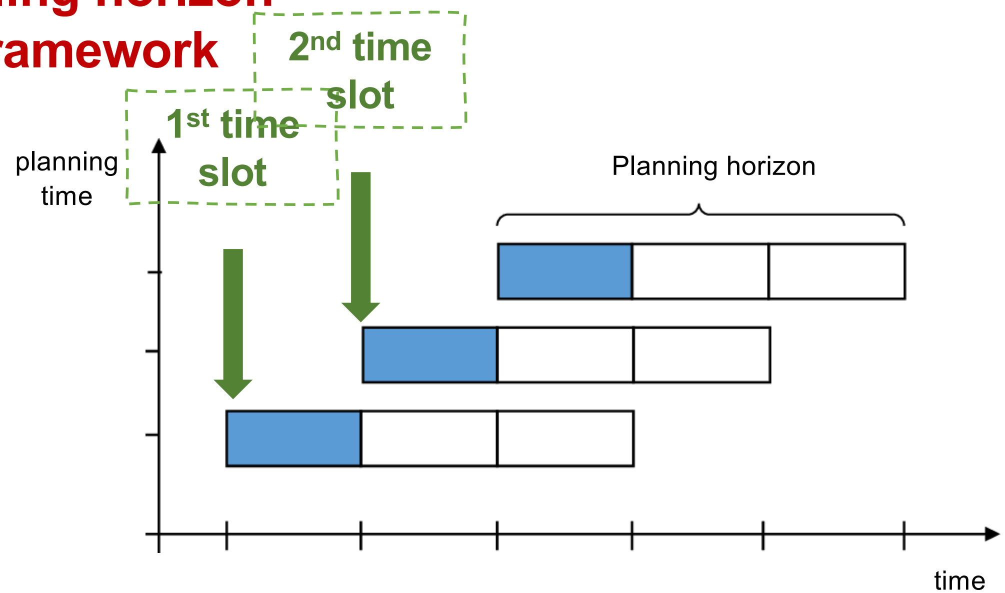
[2] **C. Yao,** S. Chen and Z. Yang, "Joint Routing and Charging Problem of Multiple Electric Vehicles: A Fast Optimization Algorithm," in IEEE Transactions on Intelligent Transportation Systems, vol. 23, no. 7, pp. 8184-8193, July 2022.

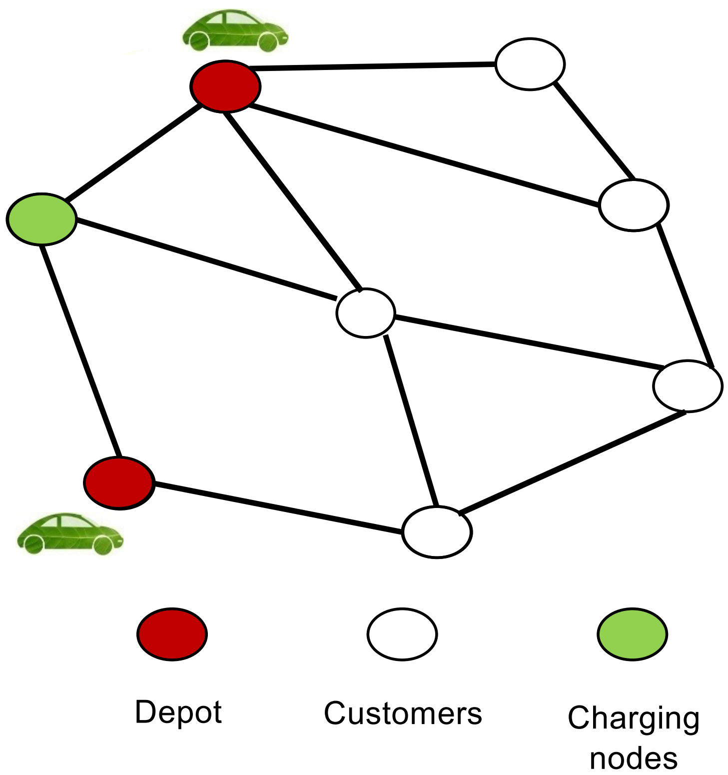
Online EV routing problem

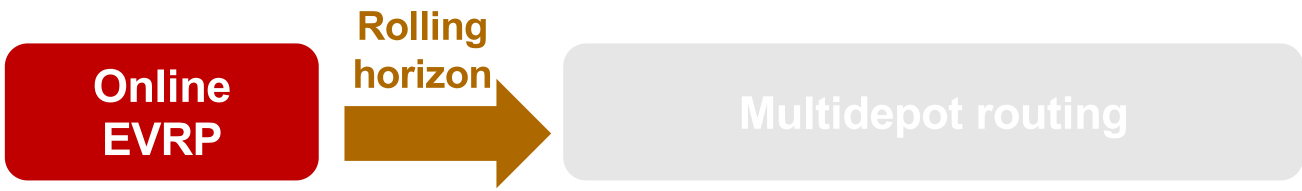
❑Online routing problem

- ➢Real-time generated customers
- ➢The uncertain charging prices

Rolling horizon framework







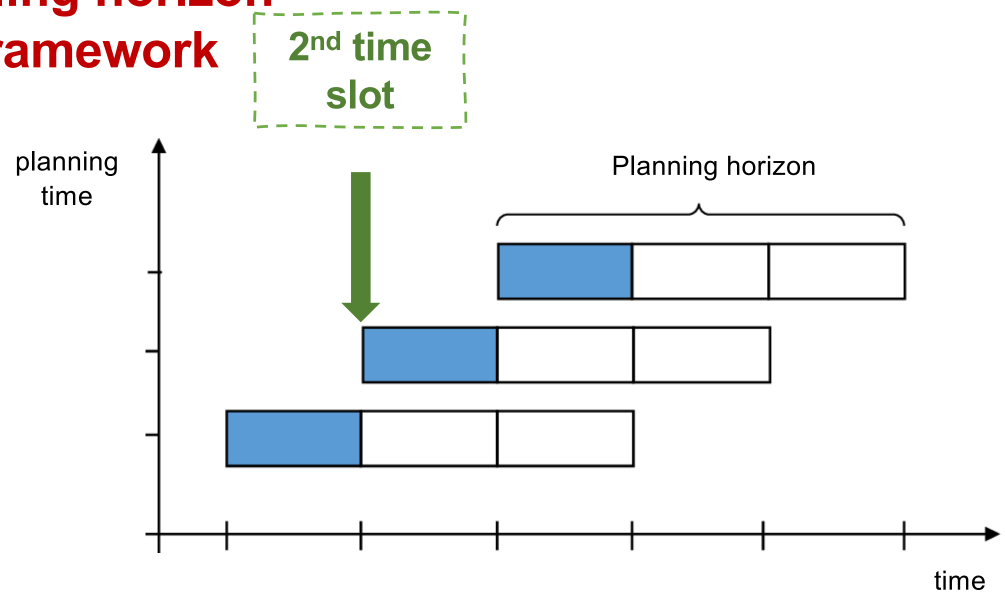
Online EV routing problem

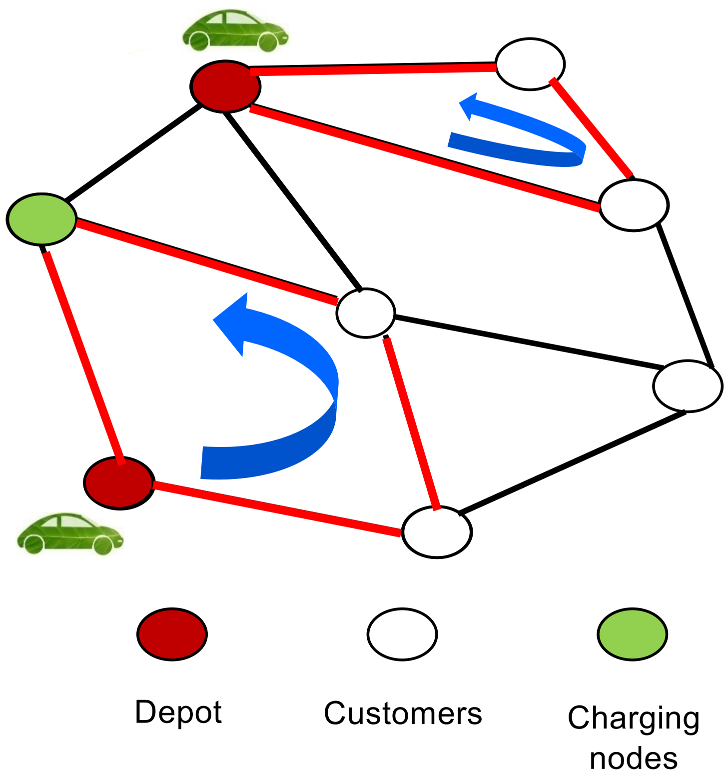
❑Online routing problem

- ➢**Real-time generated customers**
- ➢**The uncertain charging prices**

Challenges: Multi-depot routing problem

Rolling horizon framework





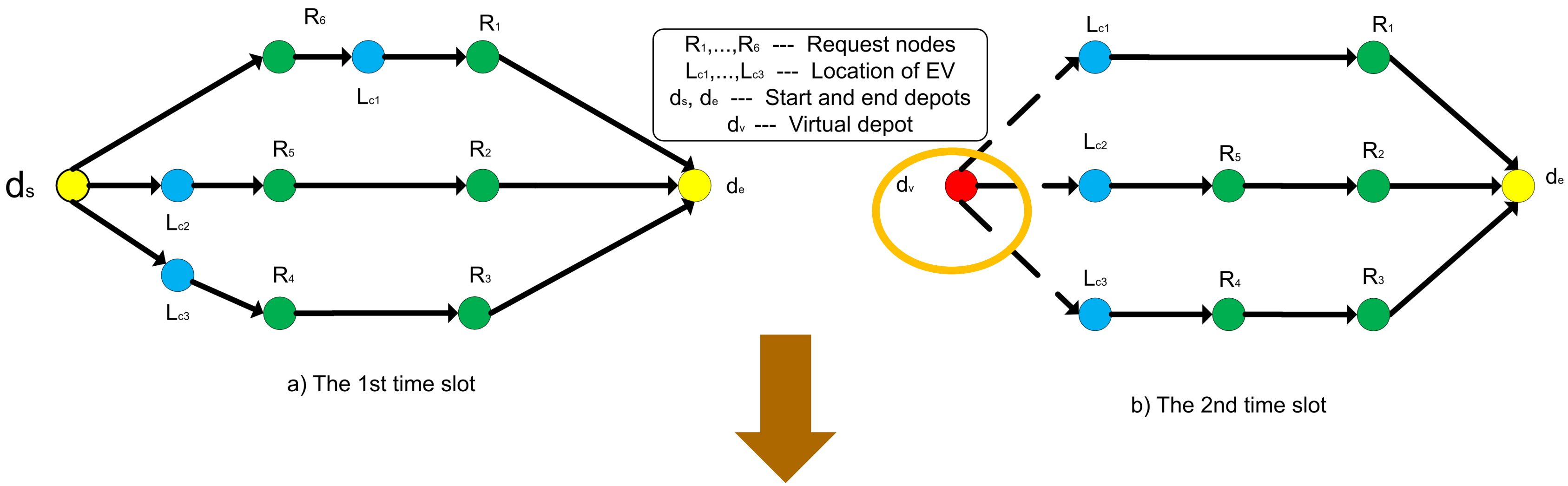
Online EVRP → Rolling horizon → **Multidepot routing**

Online EV routing problem

❑Multi-depot routing problem

➢The equivalent transformation rule

- The *road selection variable* and *travel time* of virtual roads are set as 1,0, respectively.
- The energy consumption of virtual roads is set as the difference between the battery capacity and current SoC of EV.

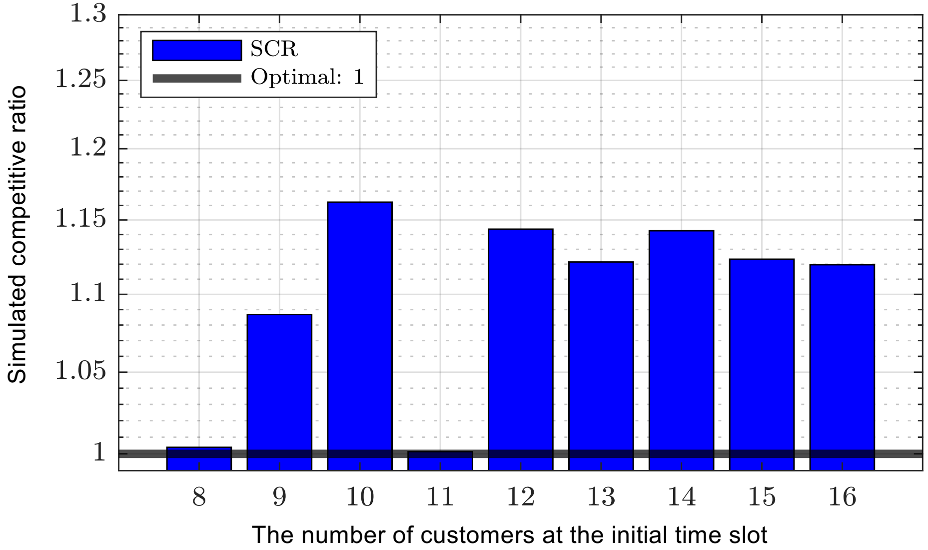


Homogenous EV at the virtual depot

Online EV routing problem

❑Simulation results

➢Evaluation of the simulated competitive ratio



The SCR of ILP is lower than **1.2**, demonstrating that the ILP works well in online routing problem

[3] **C. Yao,** S. Chen and Z. Yang, "Online Distributed Routing Problem of Electric Vehicles," in IEEE Transactions on Intelligent Transportation Systems, vol. 23, no. 9, pp. 16330-16341, Sept. 2022.

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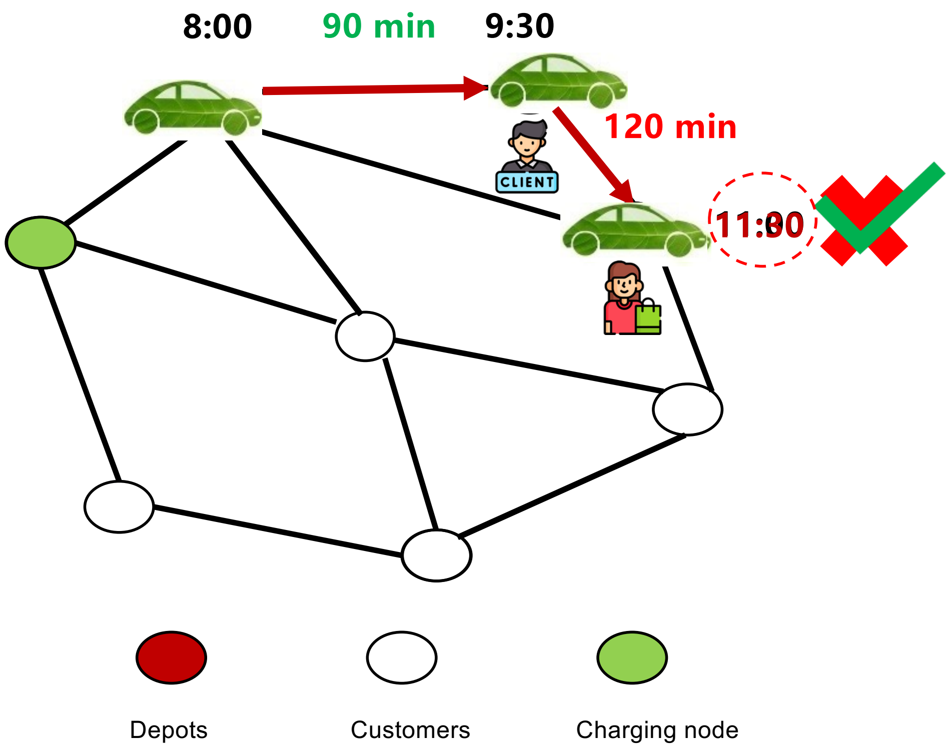
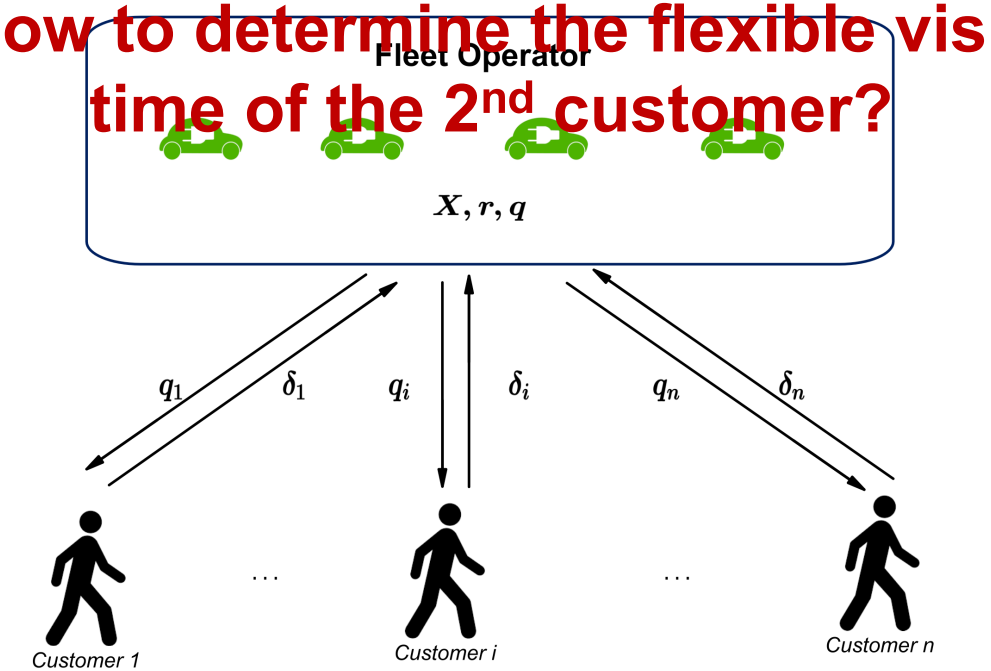
2. Electric Vehicles Routing Problem
3. **Electric Vehicles Routing Problem with Time Flexibility**
4. Conclusion

EVRP with time flexibility

❑Time flexibility



How to determine the flexible visit time of the 2nd customer?



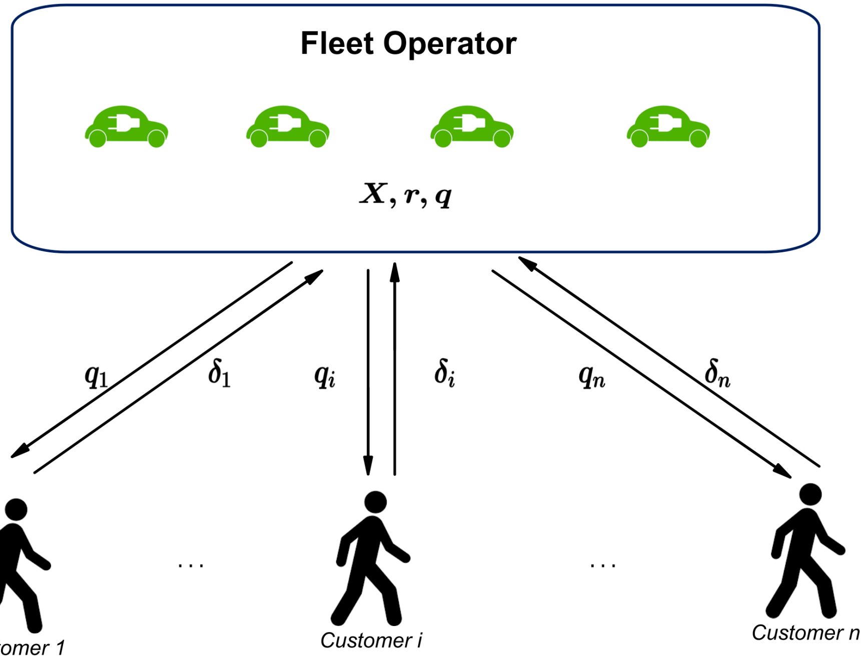
EVRP with time flexibility

❑Mathematical model of the fleet operator

➢Objective function

$$\min_{x_{ij}^k \in \mathbb{B}, r_i^k, q_j, t_j \in \mathbb{R}} \sum_{i \in \mathcal{C}} \sum_{\tau \in \Lambda} \frac{p_{i\tau} B_{i\tau} \Delta\tau}{g_i} + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} (c_i + \omega_T T_{ij} + \omega_T r_i^k g_i) x_{ij}^k$$
$$+ \sum_{j \in \mathcal{R}} q_j \delta_j^* \sum_{i \in \mathcal{V}} \sum_{k \in \mathcal{K}} x_{ij}^k$$

Operation cost of the fleet +
delivery fee discount



➢Constraint

- SoC
- Vehicle flow
- Visit time
- Flexible time window

$$t_j^L \le t_j \le t_j^L + \delta_j, \quad \forall j \in \mathcal{R}.$$

EVRP with time flexibility

❑Mathematical model of customers

$$\min_{\delta_j \in \mathbb{R}} \quad \mathcal{J}(\delta_j) - q_j\delta_j\,,$$
$$\text{s.t.} \quad 0 \le \delta_j \le \bar{\delta}_j$$

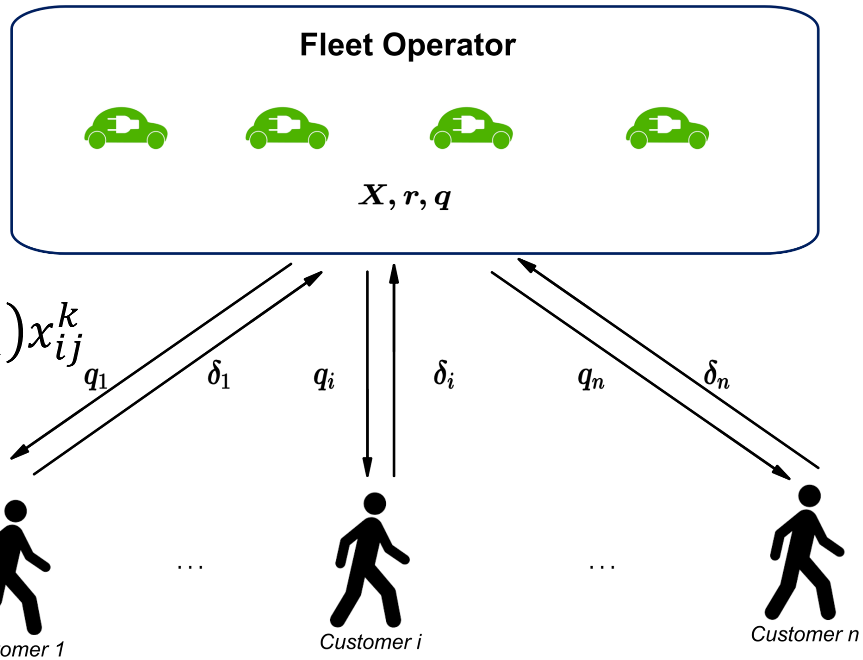
Minimize the inconvenience cost +
maximize the discount of delivery fee

❑The joint model of the fleet and customers

$$\min_X \sum_{i\in\mathcal{C}}\sum_{\tau\in\Lambda}\frac{p_{i\tau}B_{i\tau}\Delta\tau}{g_i} + \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}}(c_i + \omega_T T_{ij} + \omega_T r_i^k g_i)x_{ij}^k$$

$$+ \sum_{j\in\mathcal{R}} q_j\delta_j^* \sum_{i\in\mathcal{V}}\sum_{k\in\mathcal{K}} x_{ij}^k$$

$$\text{s.t.} \quad \delta_j^* \in \arg\min_{\delta_j}\{\mathcal{J}(\delta_j) - q_j\delta_j, 0 \le \delta_j \le \bar{\delta}_j\},$$
$$\forall j \in \mathcal{R},$$



How to solve the joint optimization problem?

EVRP with time flexibility

❑The equivalent reformulation method

➢KKT optimality condition

$$\nabla \mathcal{J}(\delta_j) - q_j - \sigma_j + u_j = 0, \forall j \in \mathcal{R}$$

Stationary condition

$$0 \le \overline{\delta}_j - \delta_j \le M\psi_j^1, \forall j \in \mathcal{R}$$
$$0 \le u_j \le M(1 - \psi_j^1), \forall j \in \mathcal{R}$$
$$0 \le \delta_j \le M\psi_j^2, \forall j \in \mathcal{R}$$
$$0 \le \sigma_j \le M(1 - \psi_j^2), \forall j \in \mathcal{R}$$

Linearized complementarity condition

Bilevel optimziation model

KKT optimality condition

Big M method, Zero duality gap

Solvable single-level problem

➢Linearization of nonlinear terms

$$\eta_{ijk}^1 \ge \omega_T r_i^k g_i - M(1 - x_{ij}^k), \forall i \in \mathcal{V}, j \in \mathcal{V}, k \in \mathcal{K}$$

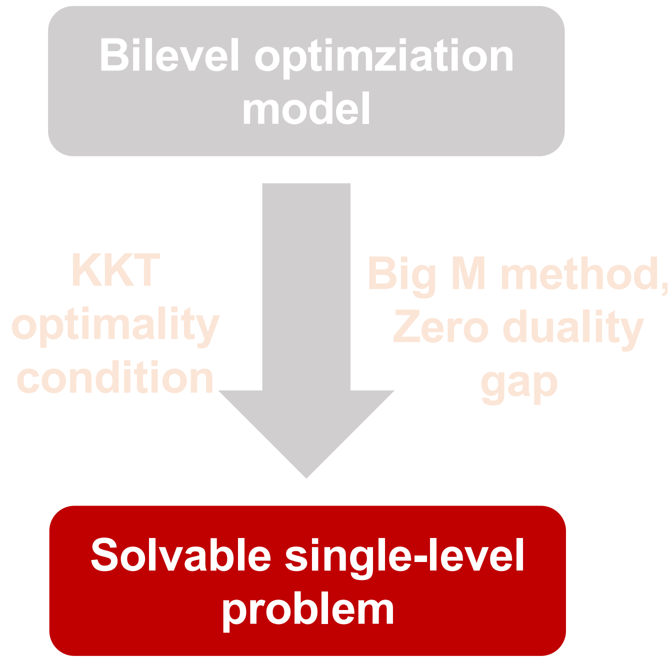
$$\eta_j^2 \ge \mathcal{J}(\delta_j) + u_j\overline{\delta}_j - \phi^*(\delta_j^*) - M\left(1 - \sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{J}} x_{ij}^k\right), \forall j \in \mathcal{R}.$$

Zero duality gap of the lower-level problem

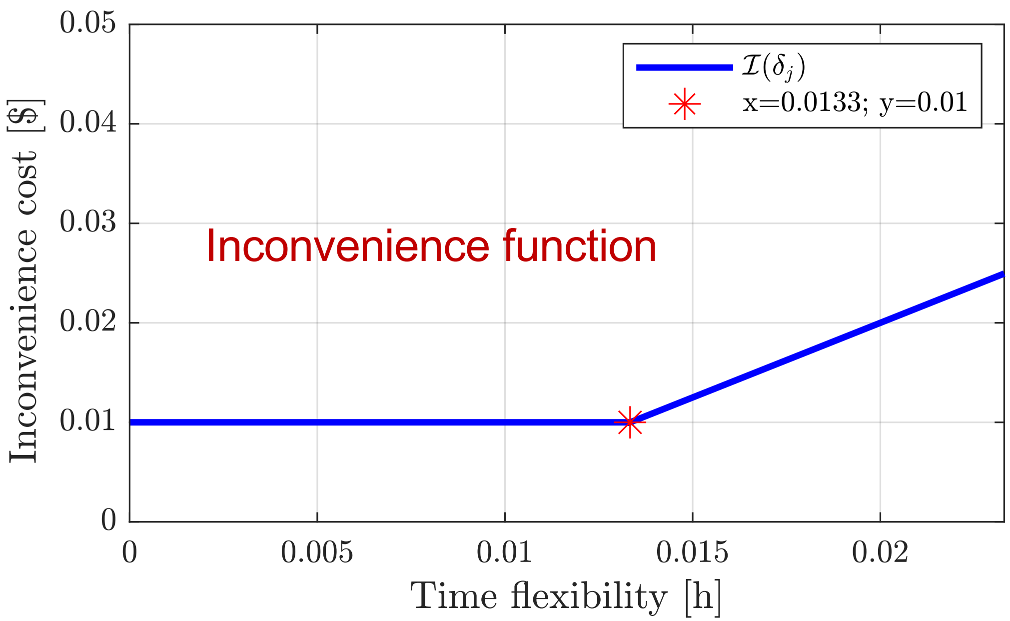
EVRP with time flexibility

❑Linearized single-level problem

- ➢**MIP-based single-level Problem**
- ➢**Solved by commercial solver**



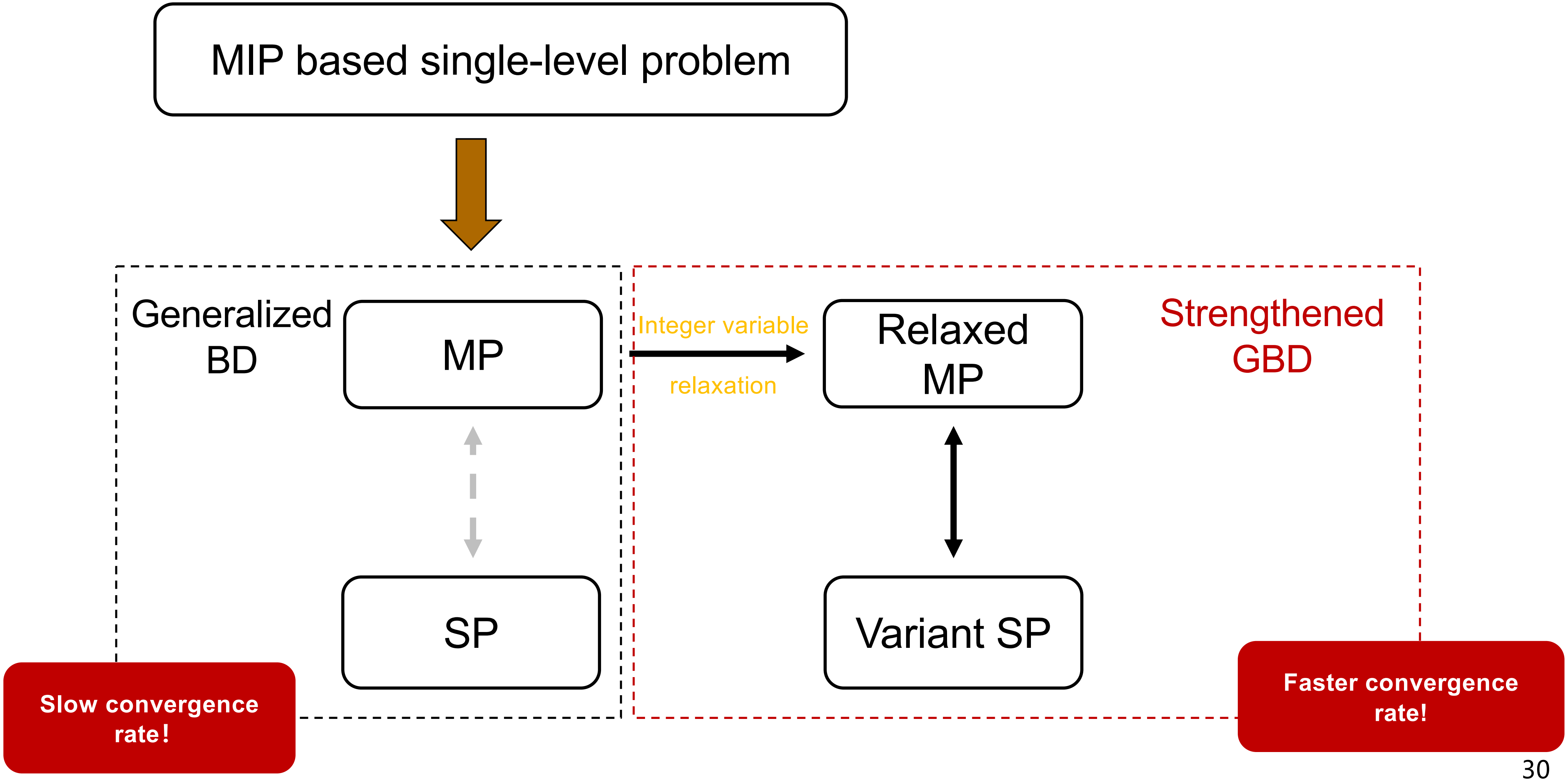
$$\min_{\mathcal{X}} \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{V}} \left(\eta^1_{ijk} + \eta^2_j + \left(\omega_T T_{ij} + c_i\right) x^k_{ij}\right)$$



How to design the distributed algorithm to protect the privacy of both sides?

EVRP with time flexibility

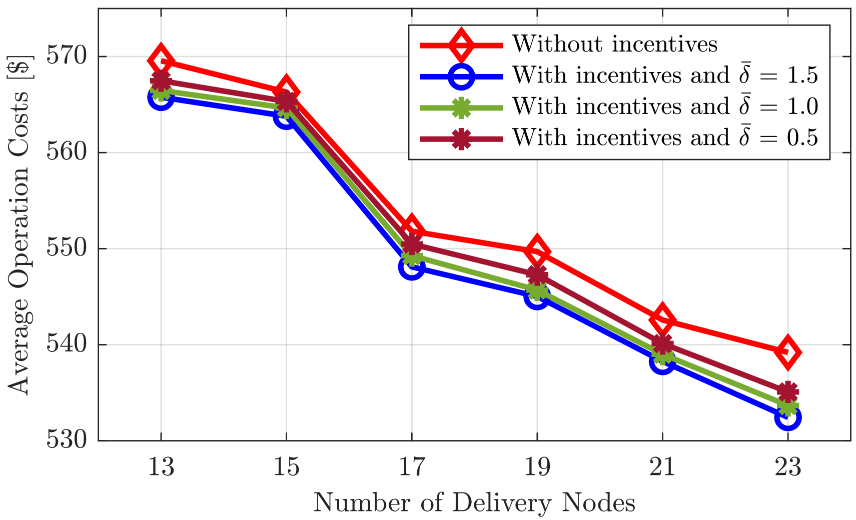
❑The algorithm details

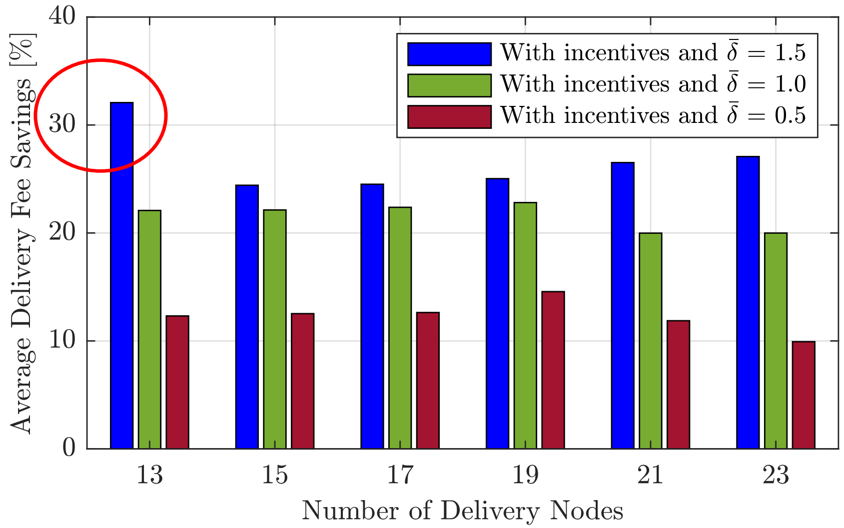


EVRP with time flexibility

❑Simulation results

➢Impact of Different Time-flexibility Levels $\bar{\delta_j}$





The operation cost becomes **smaller** with increasing values of $\bar{\delta_j}$;

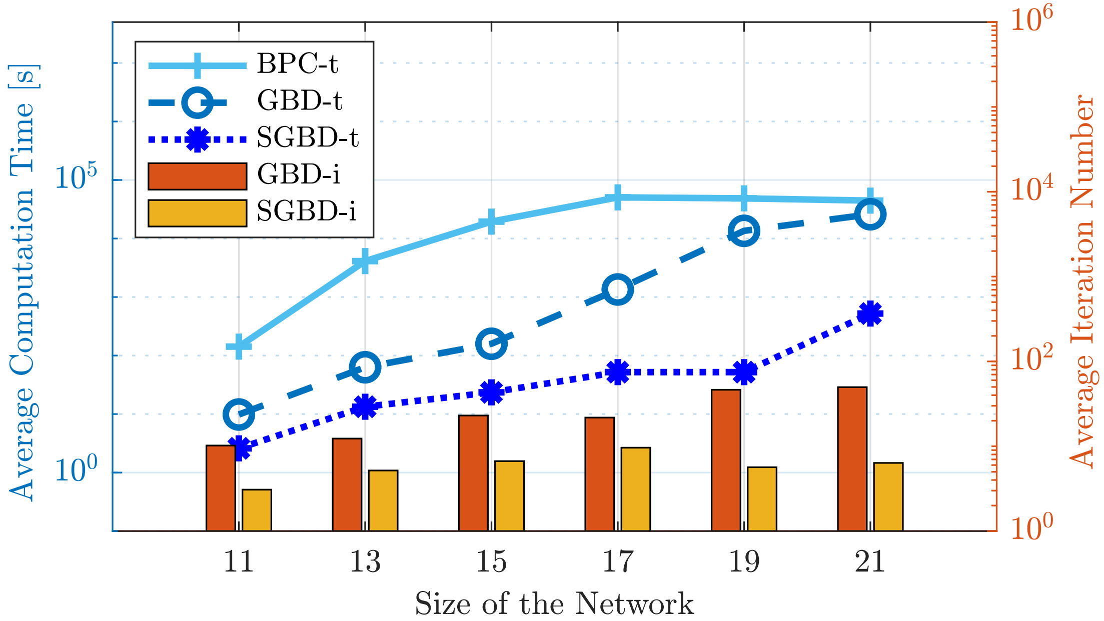
The delivery fee saving becomes **lager** with increasing values of $\bar{\delta_j}$

[5] **C. Yao,** S. Chen and Z. Yang, "Cooperative Operation of the Fleet Operator and Incentive-Aware Customers in an On-Demand Delivery System: A Bi-Level Approach," in IEEE Internet of Things Journal, doi: 10.1109/JIOT.2023.3324047.

EVRP with time flexibility

❑Simulation results

➢Comparison results between SGBD, GBD, BCP



SGBD shows a better performance than GBD, and BCP in terms of the number of iterations and computation time

[6] **C. Yao,** S. Chen, M. Salazar and Z. Yang, "Joint Routing and Charging Problem of Electric Vehicles With Incentive-Aware Customers Considering Spatio-Temporal Charging Prices," in IEEE Transactions on Intelligent Transportation Systems, vol. 24, no. 11, pp. 12215-12226, Nov. 2023.

EVRP with time flexibility

❑Simulation results

➢The scalability of SGBD

Instance	$\lvert\mathcal{V}\rvert = 11$ $\lvert\mathcal{E}\rvert = 22$	$\lvert\mathcal{V}\rvert = 51$ $\lvert\mathcal{E}\rvert = 102$	$\lvert\mathcal{V}\rvert = 101$ $\lvert\mathcal{E}\rvert = 202$	$\lvert\mathcal{V}\rvert = 151$ $\lvert\mathcal{E}\rvert = 302$
Run time (s)	2.06	121.47	837.35	$*$
ϵ	0	0.41%	2.33%	12.23%

SGBD can solve the EVRP with up to 150 nodes and 15 EVs

[6] **C. Yao,** S. Chen, M. Salazar and Z. Yang, "Joint Routing and Charging Problem of Electric Vehicles With Incentive-Aware Customers Considering Spatio-Temporal Charging Prices," in IEEE Transactions on Intelligent Transportation Systems, vol. 24, no. 11, pp. 12215-12226, Nov. 2023.

Outline

1. Research Background
2. Electric Vehicles Routing Problem
3. Electric Vehicles Routing Problem with Time Flexibility
4. **Conclusion**

Conclusion

- ❑Design the approximate method to solve large-sized EVRP in polynomial time
- ❑Devise the rolling horizon based method to solve the online EVRP with the near-optimal solution
- ❑Propose a bilevel optimization model, to characterize the mathematical model of customers and the electric vehicle fleet
- ❑Design the KKT condition based equivalent reformulation method, and the strengthened generalized Benders decomposition method

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Thank you! ***Q & A***

Acknowledgement

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